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The Influence of Nuclear Data Uncertainties on Properties of Critical Assemblies

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by

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THE INFLUENCE OF NUCLEAR DATA UNCERTAINTIES ON
PROPERTIES OF CRITICAL ASSEMBLIES

1. Introduction

In [1] the influence of the nuclear data uncertainties on the parameters of a large steam-cooled fast reactor (D-1 design [2]) was investigated. As a continuation of this study the influence of data uncertainties on the properties of small critical assemblies is examined. For the D-1 design the main safety and stability parameters (Doppler coefficient and steam density coefficient) had most attention while for the critical assemblies the quantities determined by experiments are the most interesting ones because comparison of experimental and calculated values with several cross section variations may be indicative for the quality of the group constant sets and for the most probable trend of the data variations. Fundamental mode calculations for 2 critical assemblies are performed: the SNEAK 3A-2 Uranium fueled assembly with a soft neutron spectrum (simulated steam coolant) and the ZPR 3-48 Plutonium fueled assembly with a harder neutron spectrum (sodium coolant with rather much carbon).

In this study the influence of the uncertainty of the capture cross section of U 238 and of the fission cross section of U 235 have been investigated. The results of the study may be used as helpful indications for futural improvements of group cross section sets.

2. The model for the calculations

For the fundamental mode calculations the atom densities of the reactor materials for the homogenized core are required. These atom densities are given in table 1.

For these atom densities of the homogenized core some less important reactor materials with similar cross sections are taken together (Co added to Fe, Mg added to Al, Mn added to Cr.). With the help of the NUSYS [3] program 1185 of Borgwaldt the multiplication factor k_{eff} was calculated as a function of the geometrical buckling Bg^2 (fig. 1). The group constant sets applied for these calculations are: the KFK SNEAK set [4] for the SNEAK 3A-2 assembly and the KFK-NAP set [4] for the ZPR 3-48 assembly. The geometrical bucklings for the further calculations were taken from fig. 1. For k_{eff} the best results available from quasi 2-dimensional (homogeneous) diffusion calculations (k TDS) with the same group constant sets were used. These values were taken from [5].

Table 1:

Material	Atomdensity x 10^{24} (Atoms/cm^3)	
	SNEAK 3A-2	ZPR 3-48
AL	1.2987 10^{-2}	1.09 10^{-4}
C	9.22 10^{-4}	2.0767 10^{-2}
CR	3.641 10^{-3}	2.681 10^{-3}
FE	1.2182 10^{-2}	9.985 10^{-3}
H	1.792 10^{-3}	-
MO	3.9 10^{-5}	2.06 10^{-4}
NA	-	6.231 10^{-3}
NI	1.842 10^{-3}	1.33 10^{-3}
O	1.4545 10^{-2}	-
SI	1.87 10^{-4}	-
TI	4.0 10^{-5}	-
PU^{239}	-	1.645 10^{-3}
PU^{240}	-	1.06 10^{-4}
PU^{241}	-	1.1 10^{-5}
PU^{242}	-	4.0 10^{-6}
U^{235}	2.028 10^{-3}	1.6 10^{-5}
U^{238}	8.111 10^{-3}	7.427 10^{-3}

Table 2:

Assembly	Groupconstant Set	k_{eff}	B_g^2 (cm^{-2})
SNEAK 3A-2	KFK-SNEAK	0.9840	25.49 10^{-4}
ZPR 3-48	KFK-NAP	0.9574	26.60 10^{-4}

3. Comparison of some properties of the D-1 design with SNEAK 3A-2 and ZPR 3-48 assemblies.

In order to examine the possibility of applying the results of [1] to the small assemblies, the neutron flux and adjoint flux spectra of the critical assemblies are compared with those of the D-1 design (fig. 2 and 3). In figure 3 it may be observed that the difference between the adjoint flux spectrum of the D-1 design and of the SNEAK 3A-2 assembly is considerable: the minimum of ϕ^+ in the high energy region shifts from about 20 keV for the D-1 design to about 1 MeV for SNEAK 3A-2. For this reason large differences may be expected for the reactivity effects caused by hydrogen density variation in both reactors. To analyse these differences the energy dependence of the reduced steam density coefficient $\frac{dk}{k}/\frac{d\rho}{\rho}$ is determined with the NUSYS pertubation code 2200 of Bachmann (fig. 4 and 5). From the figures 2 to 5 we may conclude:

a) The leakage component is more important for the SNEAK 3A-2 assembly than for the D-1 design (as may be expected from the reactor dimensions)

b) The quantitative results for the influence of cross sections variations in [1] can not be used for quantitative predictions of the steam density coefficient variations of SNEAK 3A-2. The more general qualitative considerations of [1] are applicable.

4. The reactor parameters considered

In recent studies of Engelmann et al [6] and of Küsters et al [5], the influence of different group cross section sets on the calculation of experimentally determined properties of the SNEAK 3A-2 assembly is investigated. Using the methods applied in [1], in the present study the influence of cross section variations on these properties is investigated more systematically. The quantities considered for SNEAK 3A-2 are:

a) k_{eff} multiplication factor

b) $\Delta K_L = K\,(N_H = 0) - K\,(N_H)$ N_H normal hydrogen concentration of SNEAK 3A-2 assembly.

c) $\Delta K_{\rho/2} = K\,(\frac{1}{2}N_H) - K\,(N_H)$

d) $\Delta K_{2\rho} = K\,(2N_H) - K\,(N_H)$

e) $$\text{R.S.D.C.} = \frac{dk}{k}/\frac{d\rho}{\rho} = \frac{K\,(1.1\,N_H) - K\,(0.9\,N_H)}{K\,(N_H)\cdot\frac{0.2\,N_H}{N_H}} = \frac{5\,K\,(1.1\,N_H) - K(0.9\,N_H)}{K\,(N_H)}$$

f) the spectral indices:

$$\frac{^{28}\sigma f}{^{25}\sigma f},\ \frac{^{49}\sigma f}{^{25}\sigma f},\ \frac{^{25}\sigma f}{^{25}\sigma f},\ \frac{^{28}\sigma\gamma}{^{25}\sigma f}\ \text{and}\ \frac{^{49}\sigma\gamma}{^{25}\sigma f}$$

calculated by the formula:

$$\frac{^{28}\sigma f}{^{25}\sigma f} = \frac{\frac{1}{N^{28}} \underset{i}{S}\ \phi_i^{28}\ \Sigma_f^i}{\frac{1}{N^{25}} \underset{i}{S}\ \phi_i^{25}\Sigma_f^i}$$

For the ZPR 3-48 assembly the following quantities were investigated:

A. K_{eff}

B. $\Delta K_L = K\ (N_{NA} = 0) - K\ (N_{NA})$ N_{NA} normal Sodium concentration of ZPR 3-48

C. The same spectral indices as for SNEAK 3A-2.

5. The nuclear data uncertainties

In the present study the same basic group constant set as in [1] is used: The KFK-SNEAK set with the weighting spectrum of the SNEAK-3A-2 assembly [4]. For some calculations for the ZPR 3-48 assembly the KFK NAP set is used. This set has the same microscopic nuclear data as the SNEAK set and the weighting spectrum of a sodium cooled reactor (NA-2 prototype design [4]). Because all criticality calculations with these sets underestimate k_{eff}, group cross section variations resulting in a larger multiplication factor are chosen. For the determination of the lower σ_γ limits of U^{238} and the upper σ_f limits of U^{235} the most recent data uncertainty evaluation of J.J. Schmidt in KFK 793 [5] is used. For U^{238} no differences with the evaluation in [1] do occur. The limits applied are given in table 3.

6. The variation of the group constants.

For the variation of the group constants on the group constant tapes the program 2229 of Bachmann is applied (See [1]). In this programm the balance between the cross sections

$$\sigma t = \sigma_f + \sigma_\gamma + \sigma e + \sigma in$$

Table 3: Cross section variations used for the calculations

Group	Energy Range	Variation of σ_f (U235) (%)	Variation of σ_γ (U238) (%)
1	6.5 - 10.5 MeV	-10 x)	-10
2	4.0 - 6.5 "	-10	-10
3	2.5 - 4.0 "	-10	-10
4	1.4 - 2.5 "	-10	-10
5	0.8 - 1.4 "	-10	-10
6	0.4 - 0.8 "	+ 7	-10
7	0.2 - 0.4 "	+ 7	-20
8	100 - 200 keV	+ 7	-20
9	46.5 - 100 "	+ 7	-20
10	21.5 - 46.5 "	+ 7	-20
11	10.0 - 21.5 "	+ 7	-20
12	4.65 - 10.0 "	+ 7	-20
13	2.15 - 4.65 "	+ 7	-20
14	1.0 - 2.15 "	+ 7	-20
15	0.456 - 1.0 "	+ 7	-15
16	215 - 465 eV	+ 7	-15
17	100 - 215 "	+ 7	-15
18	46.5 - 100 "	+ 7	-15

x) Because in group 1-5 σ_f (U235) has as upper limit the KFK-SNEAK data, the calculations are performed with the lower limit.

is usually maintained by changing σ_t if one of the other cross sections change. However, because the σ_t cross section usually is known with the best accuracy this method may lead to values of σ_t out of the uncertainty limits if other, more inaccurate cross sections are changed between their outside limits. Moreover the cross section for elastic scattering often is calculated from:

$$\sigma_e = \sigma_t - (\sigma_\gamma + \sigma_f + \sigma_{in}).$$

Therefore from the physical point of view it is better to maintain the cross section balance by changing σ_e instead of σ_t. The different methods mainly lead to changes in the transport cross section. That is why the errors may become most significant for small assemblies (with high leakage). Some comparison calculations in [1] did not show observable differences for the D-1 design. However, for the SNEAK 3A-2 assembly the differences have to be analysed more detailed because of the higher leakage component.

7. Results of the investigations

The results of the calculations are tabulated in the tables 4, 5 and 6. The calculations for the ZPR 3-48 assembly are performed with the help of the same modified KFK SNEAK set group constants as for the SNEAK 3A-2 assembly (steam coolant weighting). This means that the calculations for the ZPR assembly will be more inaccurate than for the SNEAK 3A-2 assembly. However, the trends calculated will be an indication for the effect of the cross section variations on the ZPR properties.

In table 4 it may be observed that some cross sections changes are favourable (with respect to the deviations between experiment and calculations) both for the multiplication factor, the steam density coefficient and the spectral indices, e.g. decreasing σ_γ in the energy range 50 eV - 1 keV (Gr. 15-18) and increasing of σ_f in the range 50 eV - 5 keV (Gr. 13-18). Not observable in table 5, but very probable with regard to the results of [1], decrease of σ_γ in the range 1 keV - 5 keV (Gr. 13, 14) will be favourable too.

For the decrease of σ_γ of U^{238} in the energy range 1 keV - 50 keV (Gr.10-14) a remarkable effect may be observed. Namely, k_L and $k_{/2}$ are changing in the favourable direction (more negative) while k_2 changes infavourably.

(less positive). The steam density coefficient does not change.
In order to explain these effects, which are contradictionary at first glance , the influence of group constant variations on Δk_U $\Delta k_{f/2}$ $\Delta k_{2\rho}$ and R.S.D.C. is investigated with the help of 2^{nd} order perturbation theory (see appendix A). Indeed a second order degradation component caused by the interaction of adjoint flux variation and neutron flux variation is independent of the sign of the steam density variation. This component has the same sign as the effects observed and may be of the same order of magnitude.

For the ZPR 3-48 assembly, table 5 shows that for the whole energy region considered the decrease of σ_γ of U^{238} is favourable for k_{eff} and $^{28}\sigma_\gamma/^{25}\sigma_f$, unfavourable for $^{49}\sigma_f/^{25}\sigma_f$. The influence on $^{28}\sigma_f/^{25}\sigma_f$ is small.

Finally in table 6 the effect of σ_t - variation instead of σ_e - variation for modifications of σ_f of U^{235} is given. Also for the smaller SNEAK 3A-2 assembly the error introduced by the wrong σ_t variations is very small (of the order of magnitude $k_{eff} \backsim 10^{-4}$).

Comparison with the results of the investigations for the D-1 design shows the same qualitative behaviour with group constant variations for the SNEAK 3A-2 assembly and for the D-1 design.

Table 4

SNEAK 3A-2 Assembly

Reactor parameter	KFK 793/KFK 776		Fundamental Mode Calculations *)											
			B^2=24.010^{-4}	B^2 = 25.49 10^{-4} cm^{-2}										
	Experi-ment	KFK SNEAK SET 2 - Dim.	KFK SNEAK SET	KFK SNEAK SET	σ_γ(U238) Low Gr. 1-5	σ_γ(U238) Low Gr. 6-9	σ_γ(U238) Low Gr.10-14	σ_γ(U238) Low Gr.15-18	σ_f(U235) Low Gr. 1-5	σ_f(U235) High Gr. 6-9	σ_f(U235) High Gr.10-12	σ_f(U235) High Gr.13-15	σ_f(U235) High Gr.16-18	σ_γ(FE) Low Gr. 9-14
k_{eff}	1.000	0.9886	1.0047	0.9886	0.9897	0.9995	1.0101	0.9926	0.9811	1.0017	0.9961	0.9944	0.9916	0.9934
$\Delta k_L \times 10^2$	- 7.0	- 5.91	- 6.32	- 6.55	- 6.54	- 6.13	- 7.04	- 6.91	- 6.63	- 5.85	- 6.42	- 7.00	- 6.83	- 6.17
$\Delta k_{\rho/2} \times 10^2$	- 3.8	- 3.17	- 3.30	- 3.43	- 3.43	- 3.22	- 3.54	- 3.64	- 3.48	- 3.10	- 3.35	- 3.63	- 3.62	- 3.42
$\Delta k_{2\rho} \times 10^2$	+ 4.8	+ 3.74	+ 3.89	+ 4.09	+ 4.08	+ 3.86	+ 3.97	+ 4.34	+ 4.05	+ 3.76	+ 3.96	+ 4.21	+ 4.34	+ 4.04
R.S.D.C. $\times 10^2$	+ 5.8	+ 4.70	+ 4.54	+ 4.76	+ 4.75	+ 4.46	+ 4.76	+ 5.01	+ 4.83	+ 4.31	+ 4.61	+ 4.98	+ 5.06	+ 4.72
$-\Delta k(\Delta T) \times 10^2$	-	-	1.04	1.01	1.01	1.02	0.96	0.961	1.01	1.00	1.00	1.01	1.02	1.02
$^{25}\sigma_\gamma/^{25}\sigma_f$	-	-	0.321	0.320	0.320	0.321	0.324	0.321	0.324	0.312	0.315	0.315	0.317	0.321
$^{28}\sigma_\gamma/^{25}\sigma_f$	0.130	0.137	0.139	0.138	0.137	0.131	0.124	0.136	0.140	0.135	0.136	0.137	0.138	0.136
$^{49}\sigma_\gamma/^{25}\sigma_f$	-	-	0.303	0.301	0.302	0.303	0.307	0.304	0.305	0.294	0.296	0.296	0.297	0.303
$^{28}\sigma_f/^{25}\sigma_f$	0.0338	0.0297	0.0288	0.0291	0.0291	0.0288	0.0284	0.0290	0.0295	0.0287	0.0289	0.0289	0.0290	0.0290
$^{49}\sigma_f/^{25}\sigma_f$	-	-	0.965	0.966	0.965	0.966	0.963	0.967	0.978	0.944	0.953	0.954	0.959	0.966

*) With heterogenity corrections from KFK 776

δk_{eff} = + 0.46 10^{-2}; $\delta\Delta k_L$ = - 0.3 10^{-2}; $\delta\Delta k_{\rho/2}$ = - 0.14 10^{-2}; $\Delta k_{2\rho}$ = + 0.32 10^{-2}

Table 5

ZPR 3-48 Assembly

Reactor parameter	KFK - 793		0-Dim. $B^2 = 26.60\ 10^{-4}$		0-Dim. $B^2 = 26.60\ 10^{-4}$ KFK SNEAK SET with			
	Experiment	KFK-NAP SET	KFK-NAP SET	KFK SNEAK SET	$\sigma_{\gamma_{Low}}$(U238) Gr. 1-5	$\sigma_{\gamma_{Low}}$(U238) Gr. 6-9	$\sigma_{\gamma_{Low}}$(U238) Gr.10-14	$\sigma_{\gamma_{Low}}$(U238) Gr.15-18
k_{eff}	1.000	0.979	0.9579	0.9605	0.9616	0.9713	0.9805	0.9616
$\Delta k_L \times 10^2$	-	-	- 3.69	- 3.80	- 3.80	- 3.86	- 4.16	- 3.80
$^{25}\sigma_\gamma / ^{25}\sigma_f$	-	-	0.277	0.280	0.280	0.281	0.285	0.280
$^{28}\sigma_\gamma / ^{25}\sigma_f$	0.138	0.146	0.148	0.146	0.144	0.137	0.130	0.145
$^{49}\sigma_\gamma / ^{25}\sigma_f$	-	-	0.188	0.189	0.189	0.190	0.194	0.189
$^{28}\sigma_f / ^{25}\sigma_f$	0.0307	0.030	0.031	0.031	0.031	0.030	0.030	0.031
$^{49}\sigma_f / ^{25}\sigma_f$	0.976	0.908	0.912	0.915	0.908	0.912	0.904	0.915

Table 6

SNEAK 3A-2 Assembly

Hydrogen density (Atoms/cm^3)	σ_f U235 Low Gr. 1-5			σ_f U235 High Gr. 6-9			σ_f U235 High Gr. 10-12			σ_f U235 High Gr. 13-15		
	k_{eff}		II - I	k_{eff}		II - I	k_{eff}		II - I	k_{eff}		II - I
	I σ_t var.	II σ_e var.	$\Delta k \times 10^4$	I σ_t var.	II σ_e var.	$\Delta k \times 10^4$	I σ_t var.	II σ_e var.	$\Delta k \times 10^4$	I σ_t var.	II σ_e var.	$\Delta k \times 10^4$
0	0.913 190	0.913 264	+ 0.74	0.941 610	0.941 496	- 1.14	0.930 324	0.930 286	- 0.38	0.922 756	0.922 755	- 0.01
$7.14 \; 10^{20}$	0.943 127	0.943 200	+ 0.73	0.967 482	0.967 383	- 0.99	0.959 496	0.959 461	- 0.35	0.954 903	0.954 904	+ 0.01
$17.92 \; 10^{20}$	0.976 505	0.976 576	+ 0.71	0.997 088	0.997 005	- 0.83	0.991 546	0.991 516	- 0.30	0.989 780	0.989 784	+ 0.04
$36.0 \; 10^{20}$	1.014 818	1.014 885	+ 0.67	1.031 486	1.031 421	- 0.65	1.027 929	1.027 905	- 0.24	1.028 642	1.028 648	+ 0.06

Influence of σ_t-variation instead of σ_e variation for σ_f modifications of U235. Fundamental mode calculations.

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Appendix A

If $\sigma\gamma$ of U-238 is decreased to its lower limit in the energy groups 10-14 the R.S.D.C. does not change. However a negative component is added to Δk_L, $\Delta k_{\rho/2}$, $\Delta k_{2\rho}$.

With the help of second order pertubation theory the effects are examined. In [1] the following equation was derived straight forward:

$$\frac{\Delta k_{21}}{k_2 k_1} = \frac{k_2 - k_1}{k_2 k_1} = \frac{1}{F_{12}} \int_v dv \left[- \langle \phi_2^+ \delta M \, \phi_1 \rangle + \frac{\langle \delta \nu \, \Sigma_f^T, \phi_1 \rangle \langle \phi_2^+, \chi \rangle}{k_2} \right] \qquad (1\text{-a})$$

with

$$F_{12} = \int_v dv \left[\langle \phi_2^+ \chi \rangle \langle \nu \, \Sigma_f^T, \, \phi_1 \rangle \right] \qquad (1\text{-b})$$

From this derivation it follows that the indices 1 and 2 may be changed on the left side of the equations (1-a) and (1-b).

The following variations are considered:

A. Steam density variation $\delta\rho$. Mainly the following group constant variations are introduced: $\delta \Sigma_{j \to i}$, $\delta \Sigma_s^i$, $\delta \Sigma_{tr}^i$ and δD_i in all groups.

B. Group constant variations in order to study the effect of the nuclear data uncertainties. Here only the variation of Σ_γ^i in the groups 10-14 will be considered.

Therefore we distinguish 4 cases:

1.	SNEAK SET	; steam density ρ ;		ϕ_1, ϕ_1^+, k_1
2.	SNEAK SET	; " " $\rho + \delta\rho$;	M_A	, ϕ_2, ϕ_2^+, k_2
3.	SNEAK SET with Σ_γ^i changed;	" " ρ ;	M_B	, ϕ_3, ϕ_3^+, k_3
4.	SNEAK SET with Σ_γ^i changed;	" " $\rho + \delta\rho$;	$M_A + M_B$,	ϕ_4, ϕ_4^+, k_4

and assume:

$$\begin{aligned} \phi_1 &= \phi \\ \phi_2 &= \phi + \delta\phi_A \\ \phi_3 &= \phi + \delta\phi_B \\ \phi_4 &= \phi + \delta\phi_A + \delta\phi_B + \delta\phi_{AB} \end{aligned} \qquad (2\text{-a})$$

$$\begin{aligned}
\phi_1^+ &= \phi^+ \\
\phi_2^+ &= \phi^+ + \delta\phi_A^+ \\
\phi_3^+ &= \phi^+ + \delta\phi_B^+ \\
\phi_4^+ &= \phi^+ + \delta\phi_A^+ + \delta\phi_B^+ + \delta\phi_{AB}^+
\end{aligned} \qquad (2\text{-b})$$

$$\begin{aligned}
k_1 &= k_1 \\
k_2 &= k_1 + \delta k_A \\
k_3 &= k_1 + \delta k_B \\
k_4 &= k_1 + \delta k_A + \delta k_B + \delta k_{AB}
\end{aligned} \qquad (2\text{-c})$$

The following quantitiy has to be examined:

$$\Delta k_{43} - \Delta k_{21} = (k_4 - k_3) - (k_2 - k_1) = \Delta k_{AB}$$

However, from (1-a) it follows:

$$\frac{\Delta k_{21}}{k_1 k_2} = X \qquad (3\text{-a}) \qquad \text{and} \qquad \frac{\Delta k_{43}}{k_3 k_4} = Y \qquad (3\text{-b})$$

(3-a)
(3-b)

With the help of (2-c) and neglecting terms with products $\Delta k_A \cdot \Delta k_B$ etc. we obtain:

$$\Delta k_{AB} \approx k_1^{\,2} \left(1 + \frac{2\Delta k_A}{k_1} + \frac{2\Delta k_B}{k_1}\right) (Y-X) \qquad (4)$$

In this expression X and Y have the factors F_{12} and F_{34} from (1-b).

It may easily be derived that

$$F_{34} = F_{12} \left\{ 1 + \underbrace{\frac{1}{F_{12}} \int_v dv \left[\frac{\langle \nu\Sigma_f^T, \delta\phi_B \rangle}{\langle \nu\Sigma_f^T, \phi \rangle} + \frac{\langle \delta\phi_B^+ + \delta\phi_{AB}^+, \chi \rangle}{\langle \phi^+ + \delta\phi_A^+, \chi \rangle} + \frac{\langle \nu\Sigma_f^T, \delta\phi_B \rangle \langle \delta\phi_B^+ + \phi_{AB}^+, \chi \rangle}{\langle \nu\Sigma_f^T, \phi \rangle \langle \phi^+ + \delta\phi_A^+, \chi \rangle} \right]}_{\varepsilon} \right\} \qquad (5\text{-a})$$

$$F_{34} = F_{12} \; (1 + \varepsilon) \qquad (5\text{-b})$$

For variations of Σ_γ^i only it follows from (1-a) and (3-a)

$$X = \frac{1}{F_{12}} \int_v dv \left[- \langle \phi_2^+ \, \delta M_A \, \phi_1 \rangle \right]$$

Because the indices 1 and 2 are interchangeable it is allowed to write:

$$X = \frac{1}{2F_{12}} \int_v dv \; - \left[\langle \phi_2^{\;+} \delta M_A \; \phi_1 \rangle - \langle \phi_1^{\;+} \delta M_A \; \phi_2 \rangle \right] \tag{6-a}$$

$$Y = \frac{1}{2F_{34}} \int_v dv \; - \left[\langle \phi_4^{\;+} \delta M_A \; \phi_3 \;\; - \langle \phi_3^{\;+} \delta M_A \; \phi_4 \rangle \right] \tag{6-b}$$

From (4), (5-b), and (6-a,b) follows:

$$\Delta K_{AB} = K_1^{\;2} \left(1 + \frac{2\Delta k_A}{k_1} + \frac{2\Delta k_B}{k_1}\right) \frac{1}{2F_{34}} \int_v dv \Big[-\langle \phi_4^+ \delta M_A \; \phi_3 \rangle - \langle \phi_3^+ \delta M_A \; \phi_4 \rangle +$$

$$+ (1+\mathcal{E}) \left\{ \langle \phi_2^+ \delta M_A \; \phi_1 \rangle + \langle \phi_1^+ \delta M_A \; \phi_2 \rangle \right\} \Big] \tag{7}$$

Substitution of (2-a,b) in (7) gives:

$$\Delta K_{AB} = K_1^{\;2} \left(1 + \frac{2\Delta k_A}{k_1} + \frac{2\Delta k_B}{k_1}\right) \frac{1}{F_{34}} \quad X$$

$$\begin{aligned} X \int_v dv \Big[& -\langle \phi^+ \delta M_A \delta \phi_B \rangle - \langle \delta \phi_B^+ \delta M_A \; \phi \rangle + \mathcal{E} \langle \phi^+ \delta M_A \; \phi \rangle \;\; + \\ & - \langle \delta \phi_B^+ \delta M_A \delta \phi_B \rangle - \tfrac{11}{2} \langle \delta \phi_A^+ \delta M_A \delta \; \phi_B \rangle - \tfrac{1}{2} \langle \delta \phi_B^+ \delta M_A \delta \phi_A \rangle \;\; + \\ & - \langle \phi^+ \delta M_A \delta \phi_{AB} \rangle - \langle \delta \phi_{AB}^+ \delta M_A \; \phi \rangle - \langle \delta \phi_{AB}^+ \delta M_A \delta \phi_B \rangle \;\; + \\ & - \langle \delta \; \phi_{AB}^+ \delta M_A \delta \phi_{AB} \rangle - \tfrac{1}{2} \langle \delta \phi_A^+ \delta M_A \delta \phi_{AB} \rangle - \tfrac{1}{2} \langle \delta \; \phi_{AB}^+ \delta M_A \delta \phi_A \rangle \Big] \end{aligned} \tag{8}$$

In equation (8) some components are independent of the sign of the $\delta \rho$ variation. Namely:

$$\begin{aligned} Z = K \int_v dv \Big[& - \tfrac{1}{2} \langle \delta \phi_A^+ \delta M_A \delta \phi_B \rangle - \tfrac{1}{2} \langle \delta \phi_B^+ \delta M_A \delta \phi_A \rangle \;\; + \\ & - \langle \phi^+ \delta M_A \delta \phi_{AB} \rangle - \langle \delta \phi_{AB}^+ \delta M_A \; \phi \rangle - \langle \delta \phi_{AB} \delta M_A \delta \phi_B \rangle \Big] \end{aligned} \tag{9}$$

For the calculation of the R.S.D.C. this component Z is eliminated by the calculation method.

The lower limit of Σ_γ^i of U^{238} in the groups 10-14 does not introduce a variation of the R.S.D.C. This means that for Δk_L, $\Delta k_{\rho/2}$, and $\Delta k_{2\rho}$ the component Z of equation (9) may be significant for the reactivity changes. Both Δk_L, $\Delta k_{\rho/2}$, and $\Delta k_{2\rho}$ may change in the same direction (as observed in the calculations.).

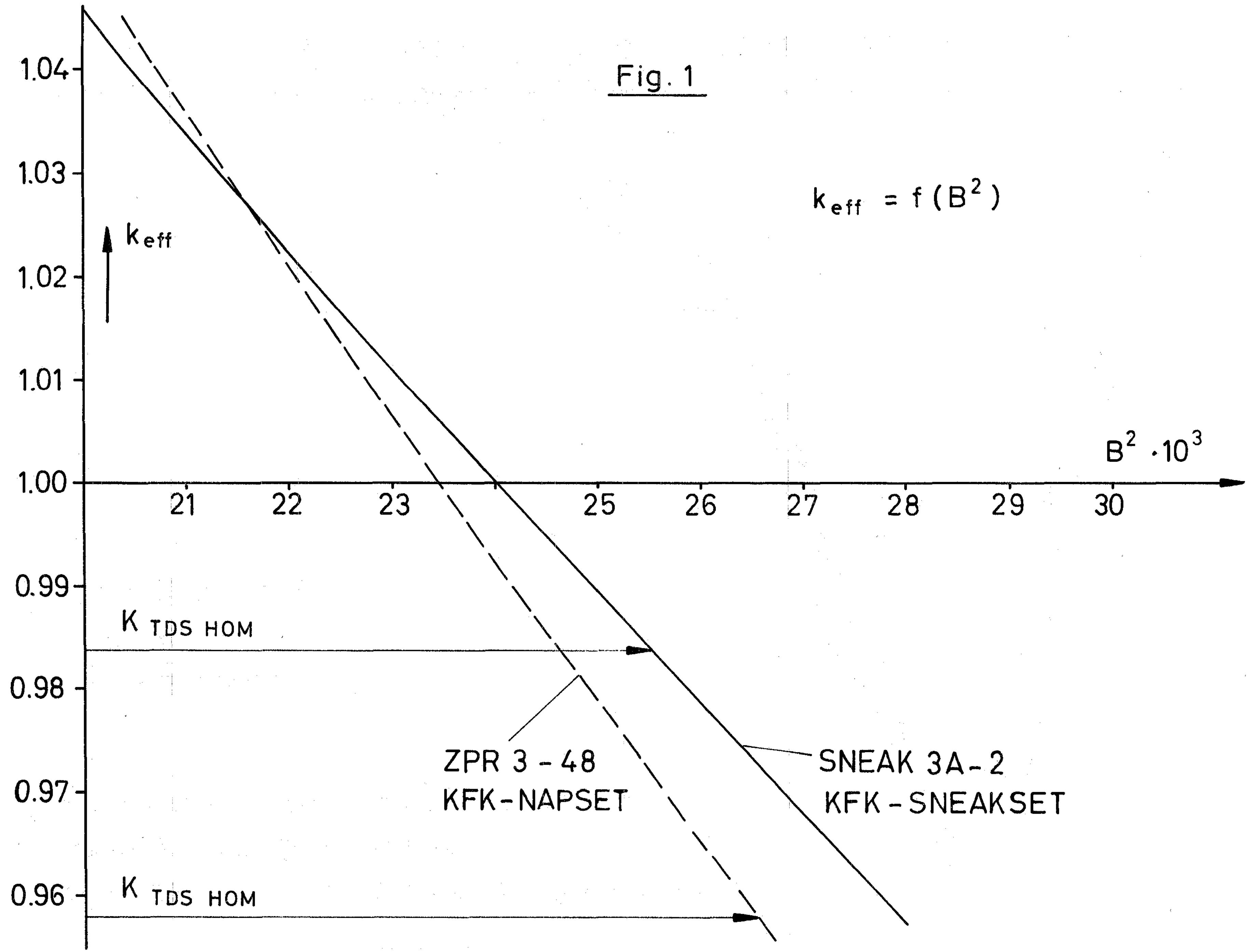
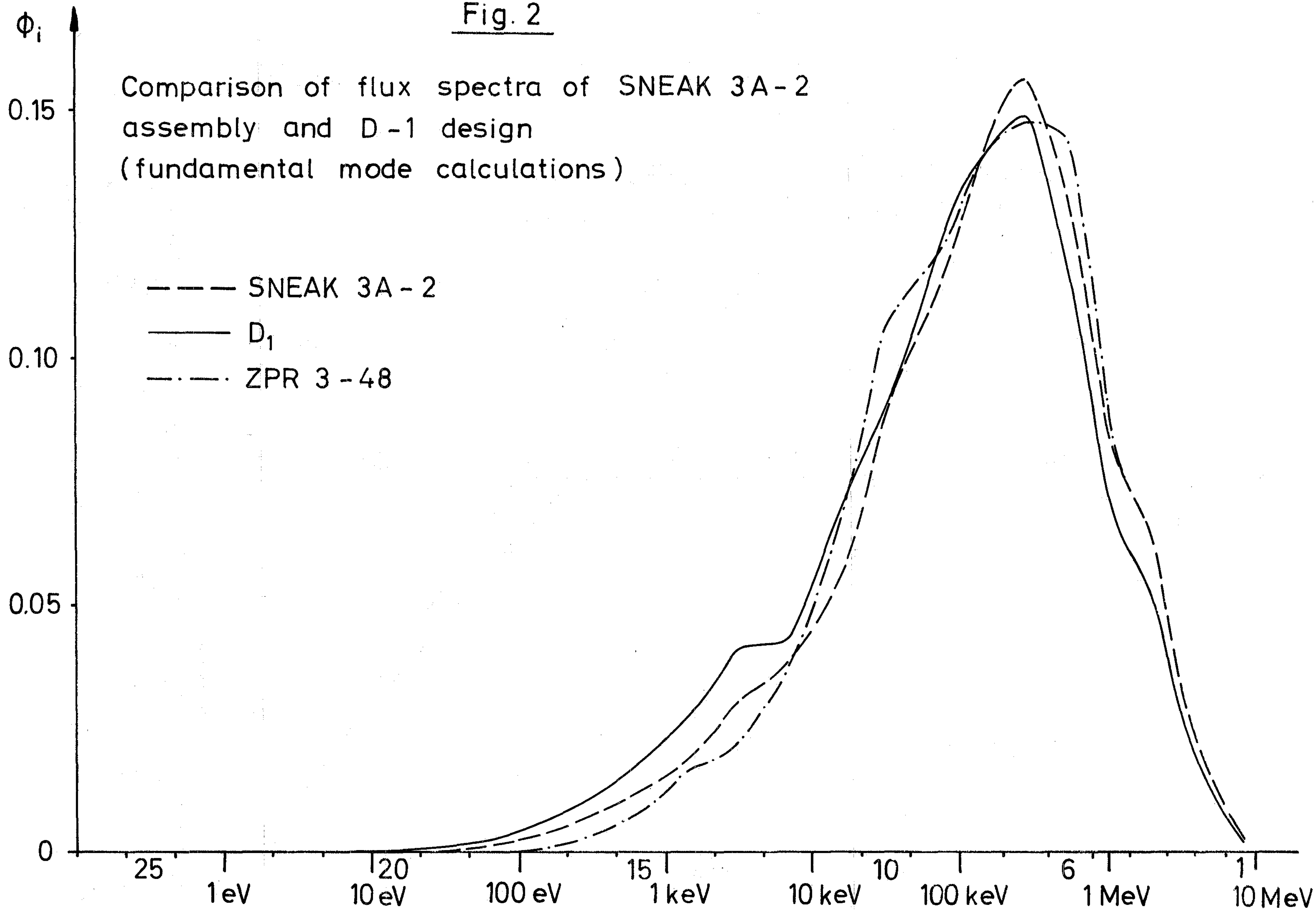
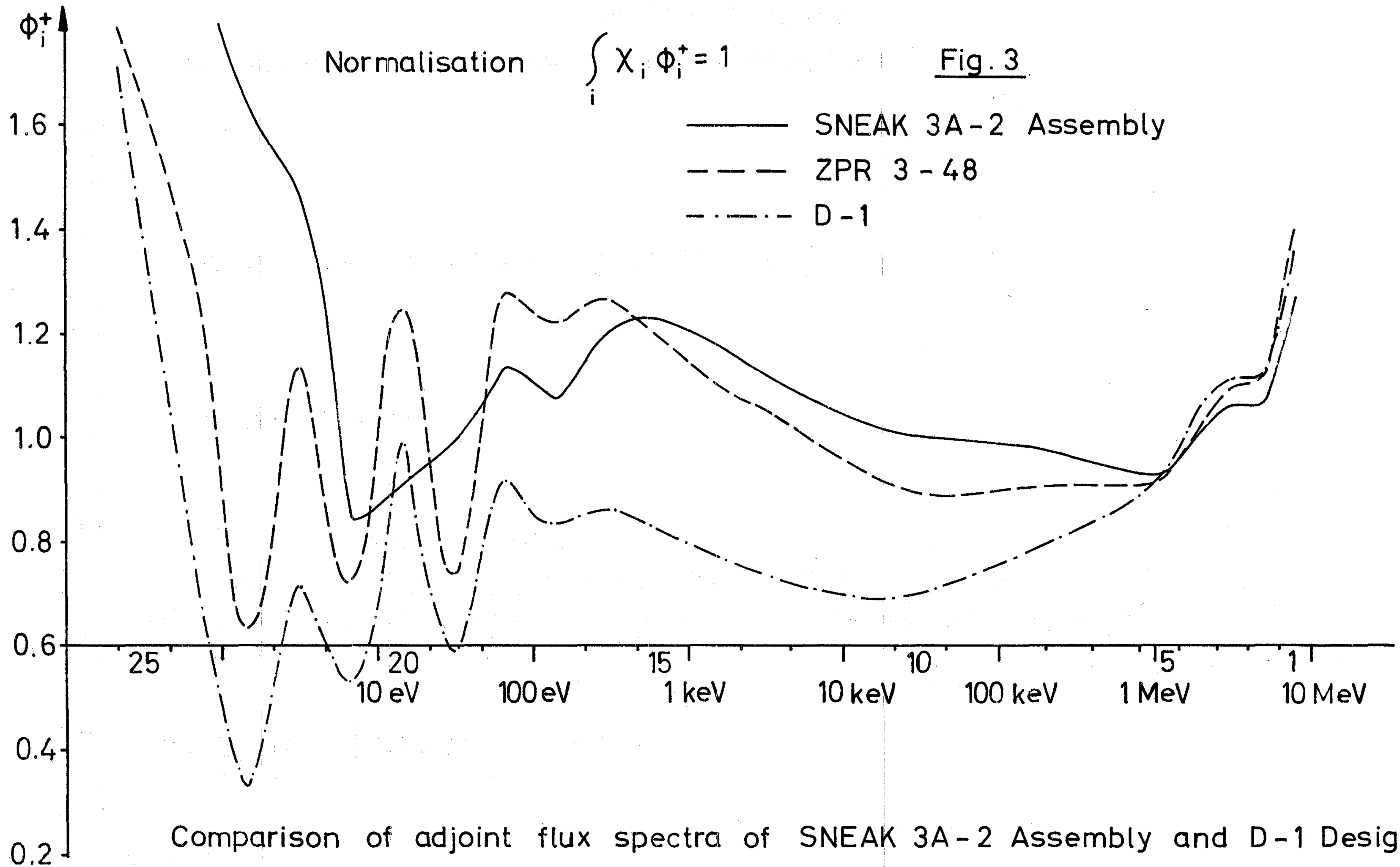


Fig. 1
$k_{eff} = f(B^2)$
k_{eff}
1.04
1.03
1.02
1.01
1.00
0.99
0.98
0.97
0.96
$B^2 \cdot 10^3$
21
22
23
25
26
27
28
29
30
K TDS HOM
K TDS HOM
ZPR 3 - 48
KFK-NAPSET
SNEAK 3A-2
KFK-SNEAKSET

Fig. 2

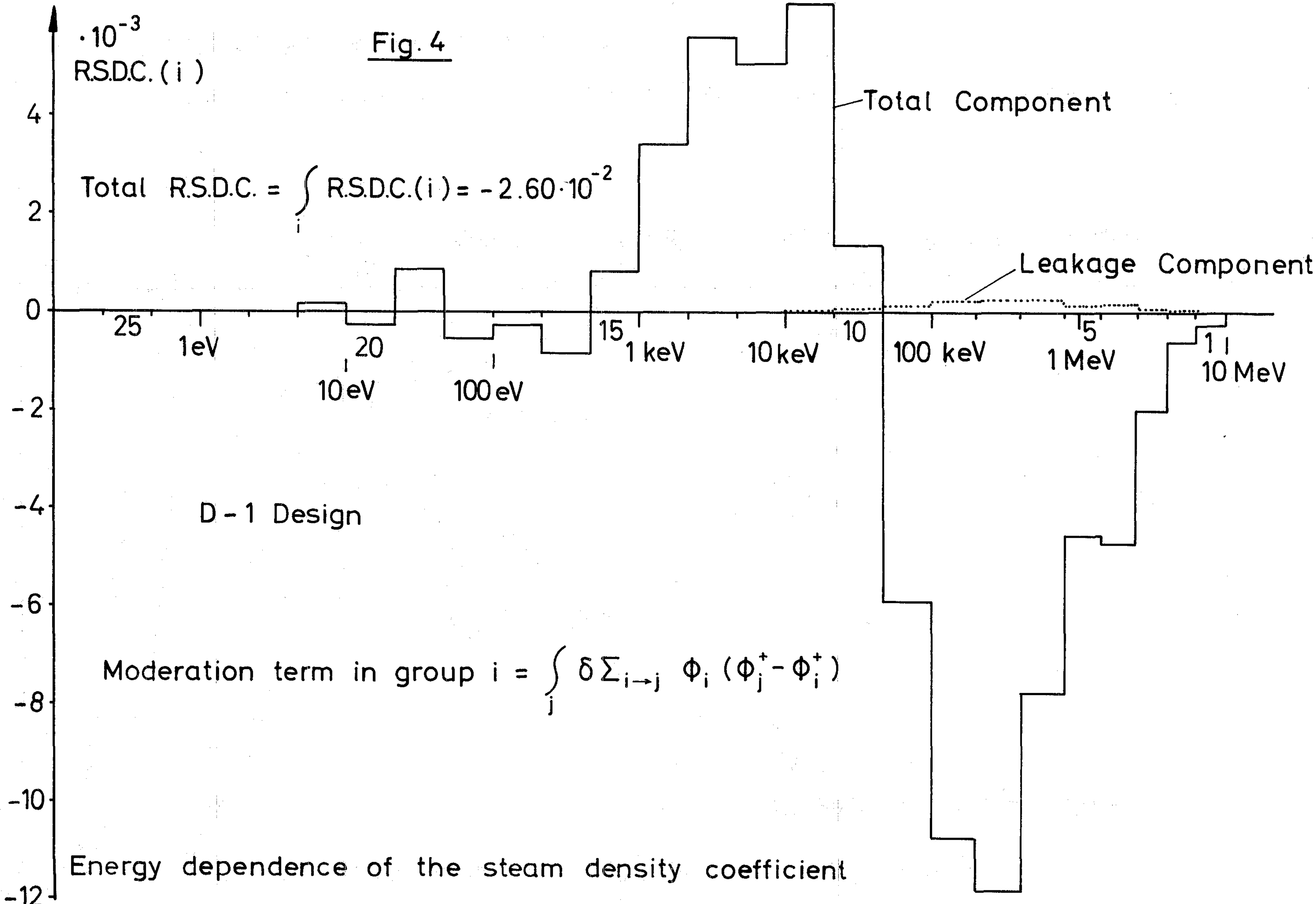


ϕ_i
Comparison of flux spectra of SNEAK 3A-2 assembly and D-1 design (fundamental mode calculations)
SNEAK 3A-2
D_1
ZPR 3-48
0.15
0.10
0.05
0
25
20
15
10
6
1
1 eV
10 eV
100 eV
1 keV
10 keV
100 keV
1 MeV
10 MeV



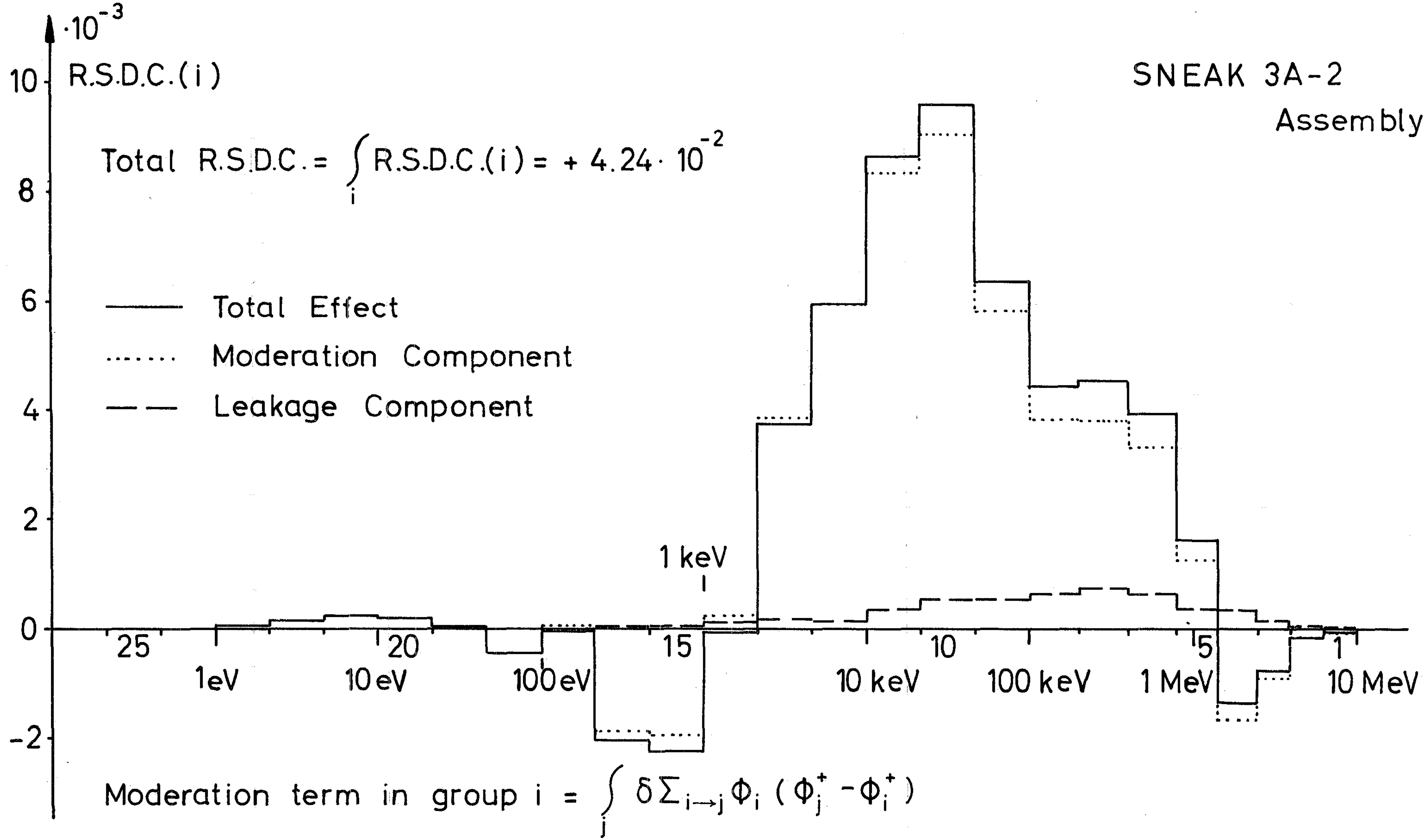
Comparison of adjoint flux spectra of SNEAK 3A-2 Assembly and D-1 Design.
(Fundamental mode calculations)

Fig. 4



Energy dependence of the steam density coefficient for the D-1 Design

Fig. 5



Energy dependence of the steam density coefficient for SNEAK 3A-2 Assembly